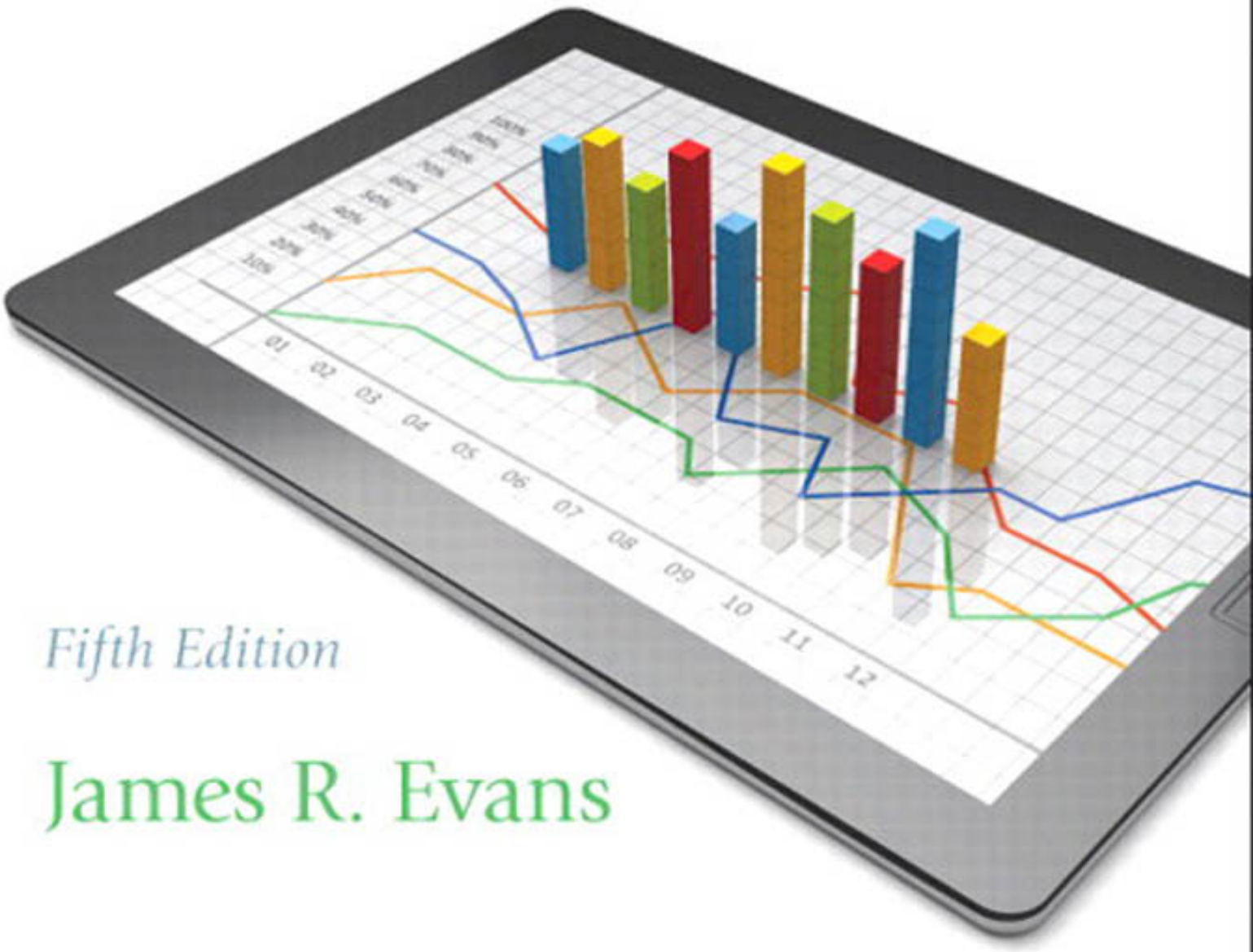
Statistics, Data Analysis, and Decision Modeling



Fifth Edition

James R. Evans

PHStat Notes

Using the *PHStat Stack Data* and *Unstack Data* Tools p. 28
One- and Two-Way Tables and Charts p. 63
Normal Probability Tools p. 97
Generating Probabilities in *PHStat* p. 98
Confidence Intervals for the Mean p. 136
Confidence Intervals for Proportions p. 136
Confidence Intervals for the Population Variance p. 137
Determining Sample Size p. 137
One-Sample Test for the Mean, Sigma Unknown p. 169
One-Sample Test for Proportions p. 169
Using Two-Sample *t*-Test Tools p. 169
Testing for Equality of Variances p. 170
Chi-Square Test for Independence p. 171
Using Regression Tools p. 209
Stepwise Regression p. 211
Best-Subsets Regression p. 212
Creating $\bar{x}$- and *R*-Charts p. 267
Creating *p*-Charts p. 268
Using the *Expected Monetary Value* Tool p. 375

Excel Notes

Creating Charts in Excel 2010 p. 29
Creating a Frequency Distribution and Histogram p. 61
Using the Descriptive Statistics Tool p. 61
Using the Correlation Tool p. 62
Creating Box Plots p. 63
Creating PivotTables p. 63
Excel-Based Random Sampling Tools p. 134
Using the *VLOOKUP* Function p. 135
Sampling from Probability Distributions p. 135
Single-Factor Analysis of Variance p. 171
Using the Trendline Option p. 209
Using Regression Tools p. 209
Using the Correlation Tool p. 211
Forecasting with Moving Averages p. 243
Forecasting with Exponential Smoothing p. 243
Using *CB Predictor* p. 244
Creating Data Tables p. 298
Data Table Dialog p. 298
Using the *Scenario Manager* p. 298
Using *Goal Seek* p. 299
Net Present Value and the NPV Function p. 299
Using the IRR Function p. 375

Crystal Ball Notes

Customizing *Define Assumption* p. 338
Sensitivity Charts p. 339
Distribution Fitting with Crystal Ball p. 339
Correlation Matrix Tool p. 341
Tornado Charts p. 341
Bootstrap Tool p. 342

TreePlan Note

Constructing Decision Trees in Excel p. 376

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Useful Statistical Functions in Excel 2010	*Description*
AVERAGE(*data range*)	Computes the average value (arithmetic mean) of a set of data.
BINOM.DIST(*number_s, trials, probability_s, cumulative*)	Returns the individual term binomial distribution.
BINOM.INV(*trials, probability_s, alpha)*	Returns the smallest value for which the cumulative binomial distribution is greater than or equal to a criterion value.
CHISQ.DIST(*x, deg_freedom, cumulative*)	Returns the left-tailed probability of the chi-square distribution.
CHISQ.DIST.RT(*x, deg_freedom, cumulative*)	Returns the right-tailed probability of the chi-square distribution.
CHISQ.TEST(*actual_range, expected_range*)	Returns the test for independence; the value of the chi-square distribution and the appropriate degrees of freedom.
CONFIDENCE.NORM(*alpha, standard_dev, size*)	Returns the confidence interval for a population mean using a normal distribution.
CONFIDENCE.T(*alpha, standard_dev, size*)	Returns the confidence interval for a population mean using a *t*-distribution.
CORREL(*arrayl, array2*)	Computes the correlation coefficient between two data sets.
EXPON.DIST(*x, lambda, cumulative*)	Returns the exponential distribution.
F.DIST(*x. deg_freedom1, deg_freedom2, cumulative*)	Returns the left-tailed *F*-probability distribution value.
F.DIST.RT(*x. deg_freedom1, deg_freedom2, cumulative*)	Returns the left-tailed *F*-probability distribution value.
FORECAST(*x, known_y's, known_x's*)	Calculates a future value along a linear trend.
GROWTH(*known_y's, known_x's, new_x's, constant)*	Calculates predicted exponential growth.
LINEST*(known_y's, known_x's, new_x's, constant, stats*)	Returns an array that describes a straight line that best fits the data.
LOGNORM.DIST(*x, mean, standard_deviation*)	Returns the cumulative lognormal distribution of *x*, where ln (*x*) is normally distributed with parameters mean and standard deviation.
MEDIAN(*data range*)	Computes the median (middle value) of a set of data.
MODE.MULT(*data range*)	Computes the modes (most frequently occurring values) of a set of data.
MODE.SNGL(*data range*)	Computes the mode of a set of data.
NORM.DIST(*x, mean, standard_dev, cumulative*)	Returns the normal cumulative distribution for the specified mean and standard deviation.
NORM.INV(*probability, mean, standard_dev*)	Returns the inverse of the cumulative normal distribution.
NORM.S.DIST(*z*)	Returns the standard normal cumulative distribution (mean = 0, standard deviation = 1).
NORM.S.INV(*probability*)	Returns the inverse of the standard normal distribution.
PERCENTILE.EXC(*array, k*)	Computes the *k*th percentile of data in a range, exclusive.
PERCENTILE.INC(*array, k*)	Computes the *k*th percentile of data in a range, inclusive.
POISSON.DIST(*x, mean, cumulative*)	Returns the Poisson distribution.
QUARTILE(*array, quart*)	Computes the quartile of a distribution.
SKEW(*data range*)	Computes the skewness, a measure of the degree to which a distribution is not symmetric around its mean.
STANDARDIZE(*x, mean, standard_deviation*)	Returns a normalized value for a distribution characterized by a mean and standard deviation.
STDEV.S(*data range*)	Computes the standard deviation of a set of data, assumed to be a sample.
STDEV.P(*data range*)	Computes the standard deviation of a set of data, assumed to be an entire population.
TREND(*known_y's, known_x's, new_x's, constant*)	Returns values along a linear trend line.
T.DIST(*x, deg_freedom, cumulative*)	Returns the left-tailed *t*-distribution value.
T.DIST.2T(*x, deg_freedom*)	Returns the two-tailed *t*-distribution value.
T.DIST.RT(*x, deg_freedom*)	Returns the right-tailed *t*-distribution.
T.INV(*probability, deg_freedom*)	Returns the left-tailed inverse of the *t*-distribution.
T.INV.2T(*probability, deg_freedom*)	Returns the two-tailed inverse of the *t*-distribution.
T.TEST(*arrayl, array2, tails, type*)	Returns the probability associated with a *t*-test.
VAR.S(*data range*)	Computes the variance of a set of data, assumed to be a sample.
VAR.P(*data range*)	Computes the variance of a set of data, assumed to be an entire population.
Z.TEST(*array, x, sigma*)	Returns the two-tailed *p*-value of a *z*-test.

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Fifth Edition

STATISTICS, DATA ANALYSIS, AND DECISION MODELING

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To Beverly, Kristin, and Lauren, the three special women in my life.

—James R. Evans

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BRIEF CONTENTS

PART I Statistics and Data Analysis 1

Chapter 1 Data and Business Decisions 3
Chapter 2 Descriptive Statistics and Data Analysis 31
Chapter 3 Probability Concepts and Distributions 65
Chapter 4 Sampling and Estimation 99
Chapter 5 Hypothesis Testing and Statistical Inference 138
Chapter 6 Regression Analysis 172
Chapter 7 Forecasting 213
Chapter 8 Introduction to Statistical Quality Control 248

PART II Decision Modeling and Analysis 269

Chapter 9 Building and Using Decision Models 271
Chapter 10 Decision Models with Uncertainty and Risk 300
Chapter 11 Decisions, Uncertainty, and Risk 343
Chapter 12 Queues and Process Simulation Modeling 378
Chapter 13 Linear Optimization 411
Chapter 14 Integer, Nonlinear, and Advanced Optimization Methods 458

Appendix 509

Index 521

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CONTENTS

Preface xxi

Part I STATISTICS AND DATA ANALYSIS 1

Chapter 1 DATA AND BUSINESS DECISIONS 3

Introduction 4
Data in the Business Environment 4
Sources and Types of Data 6
Metrics and Data Classification 7
Statistical Thinking 11
Populations and Samples 12
Using Microsoft Excel 13
Basic Excel Skills 14
Skill-Builder Exercise 1.1 14
Copying Formulas and Cell References 14
Skill-Builder Exercise 1.2 15
Functions 16
Skill-Builder Exercise 1.3 18
Other Useful Excel Tips 18
Excel Add-Ins 19
Skill-Builder Exercise 1.4 20
Displaying Data with Excel Charts 21
Column and Bar Charts 21
Skill-Builder Exercise 1.5 22
Line Charts 23
Skill-Builder Exercise 1.6 23
Pie Charts 23
Skill-Builder Exercise 1.7 23
Area Charts 24
Scatter Diagrams 24
Skill-Builder Exercise 1.8 24
Miscellaneous Excel Charts 25
Ethics and Data Presentation 25
Skill-Builder Exercise 1.9 26
Basic Concepts Review Questions 27
Problems and Applications 27
Case: A Data Collection and Analysis Project 28

Chapter 2 DESCRIPTIVE STATISTICS AND DATA ANALYSIS 31
Introduction 32
Descriptive Statistics 32
Frequency Distributions, Histograms, and Data Profiles 33
Categorical Data 34
Numerical Data 34
Skill-Builder Exercise 2.1 38
Skill-Builder Exercise 2.2 38
Data Profiles 38
Descriptive Statistics for Numerical Data 39
Measures of Location 39
Measures of Dispersion 40
Skill-Builder Exercise 2.3 42
Measures of Shape 43
Excel Descriptive Statistics Tool 44
Skill-Builder Exercise 2.4 44
Measures of Association 45
Skill-Builder Exercise 2.5 47
Descriptive Statistics for Categorical Data 47
Skill-Builder Exercise 2.6 48
Visual Display of Statistical Measures 49
Box Plots 49
Dot-Scale Diagrams 49
Skill-Builder Exercise 2.7 49
Outliers 50
Data Analysis Using PivotTables 50
Skill-Builder Exercise 2.8 53
Skill-Builder Exercise 2.9 53
Basic Concepts Review Questions 54
Problems and Applications 54
Case: The Malcolm Baldrige Award 57
Skill-Builder Exercise 2.10 59
Skill-Builder Exercise 2.11 60

Chapter 3 PROBABILITY CONCEPTS AND DISTRIBUTIONS 65
Introduction 66
Basic Concepts of Probability 66
Basic Probability Rules and Formulas 67
Conditional Probability 68
Skill-Builder Exercise 3.1 70
Random Variables and Probability Distributions 70
Discrete Probability Distributions 73
Expected Value and Variance of a Discrete Random Variable 74

Skill-Builder Exercise 3.2 75
Bernoulli Distribution 75
Binomial Distribution 75
Poisson Distribution 76
Skill-Builder Exercise 3.3 78
Continuous Probability Distributions 78
Uniform Distribution 80
Normal Distribution 81
Skill-Builder Exercise 3.4 84
Triangular Distribution 84
Exponential Distribution 85
Probability Distributions in *PHStat* 86
Other Useful Distributions 86
Joint and Marginal Probability Distributions 89
Basic Concepts Review Questions 90
Problems and Applications 90
Case: Probability Analysis for Quality Measurements 94

Chapter 4 SAMPLING AND ESTIMATION 99
Introduction 100
Statistical Sampling 100
Sample Design 100
Sampling Methods 101
Errors in Sampling 103
Random Sampling From Probability Distributions 103
Sampling From Discrete Probability Distributions 104
Skill-Builder Exercise 4.1 105
Sampling From Common Probability Distributions 105
A Statistical Sampling Experiment in Finance 106
Skill-Builder Exercise 4.2 106
Sampling Distributions and Sampling Error 107
Skill-Builder Exercise 4.3 110
Applying the Sampling Distribution of the Mean 110
Sampling and Estimation 110
Point Estimates 111
Unbiased Estimators 112
Skill-Builder Exercise 4.4 113
Interval Estimates 113
Confidence Intervals: Concepts and Applications 113
Confidence Interval for the Mean with Known Population Standard Deviation 114
Skill-Builder Exercise 4.5 116

Confidence Interval for the Mean with Unknown Population Standard Deviation 116
Confidence Interval for a Proportion 118
Confidence Intervals for the Variance and Standard Deviation 119
Confidence Interval for a Population Total 121
Using Confidence Intervals for Decision Making 122
Confidence Intervals and Sample Size 122
Prediction Intervals 124
Additional Types of Confidence Intervals 125
Differences Between Means, Independent Samples 125
Differences Between Means, Paired Samples 125
Differences Between Proportions 126
Basic Concepts Review Questions 126
Problems and Applications 126
Case: Analyzing a Customer Survey 129
Skill-Builder Exercise 4.6 131
Skill-Builder Exercise 4.7 132
Skill-Builder Exercise 4.8 133
Skill-Builder Exercise 4.9 133

Chapter 5 HYPOTHESIS TESTING AND STATISTICAL INFERENCE 138
Introduction 139
Basic Concepts of Hypothesis Testing 139
Hypothesis Formulation 140
Significance Level 141
Decision Rules 142
Spreadsheet Support for Hypothesis Testing 145
One-Sample Hypothesis Tests 145
One-Sample Tests for Means 145
Using *p*-Values 147
One-Sample Tests for Proportions 148
One Sample Test for the Variance 150
Type II Errors and the Power of A Test 151
Skill-Builder Exercise 5.1 153
Two-Sample Hypothesis Tests 153
Two-Sample Tests for Means 153
Two-Sample Test for Means with Paired Samples 155
Two-Sample Tests for Proportions 155
Hypothesis Tests and Confidence Intervals 156
Test for Equality of Variances 157
Skill-Builder Exercise 5.2 158
Anova: Testing Differences of Several Means 158
Assumptions of ANOVA 160
Tukey–Kramer Multiple Comparison Procedure 160

Chi-Square Test for Independence 162
Skill-Builder Exercise 5.3 164
Basic Concepts Review Questions 164
Problems and Applications 164
Case: *HATCO, Inc.* 167
Skill-Builder Exercise 5.4 169

Chapter 6 REGRESSION ANALYSIS 172
Introduction 173
Simple Linear Regression 174
Skill-Builder Exercise 6.1 175
Least-Squares Regression 176
Skill-Builder Exercise 6.2 178
A Practical Application of Simple Regression to Investment Risk 178
Simple Linear Regression in Excel 179
Skill-Builder Exercise 6.3 180
Regression Statistics 180
Regression as Analysis of Variance 181
Testing Hypotheses for Regression Coefficients 181
Confidence Intervals for Regression Coefficients 182
Confidence and Prediction Intervals for *X*-Values 182
Residual Analysis and Regression Assumptions 182
Standard Residuals 184
Skill-Builder Exercise 6.4 184
Checking Assumptions 184
Multiple Linear Regression 186
Skill-Builder Exercise 6.5 186
Interpreting Results from Multiple Linear Regression 188
Correlation and Multicollinearity 188
Building Good Regression Models 190
Stepwise Regression 193
Skill-Builder Exercise 6.6 193
Best-Subsets Regression 193
The Art of Model Building in Regression 194
Regression with Categorical Independent Variables 196
Categorical Variables with More Than Two Levels 199
Skill-Builder Exercise 6.7 201
Regression Models with Nonlinear Terms 201
Skill-Builder Exercise 6.8 202
Basic Concepts Review Questions 204
Problems and Applications 204
Case: *Hatco* 207

Chapter 7 FORECASTING 213
Introduction 214
Qualitative and Judgmental Methods 214
Historical Analogy 215
The Delphi Method 215
Indicators and Indexes for Forecasting 215
Statistical Forecasting Models 216
Forecasting Models for Stationary Time Series 218
Moving Average Models 218
Error Metrics and Forecast Accuracy 220
Skill-Builder Exercise 7.1 222
Exponential Smoothing Models 222
Skill-Builder Exercise 7.2 224
Forecasting Models for Time Series with a Linear Trend 224
Regression-Based Forecasting 224
Advanced Forecasting Models 225
Autoregressive Forecasting Models 226
Skill-Builder Exercise 7.3 228
Forecasting Models with Seasonality 228
Incorporating Seasonality in Regression Models 229
Skill-Builder Exercise 7.4 231
Forecasting Models with Trend and Seasonality 231
Regression Forecasting with Causal Variables 231
Choosing and Optimizing Forecasting Models Using *CB Predictor* 233
Skill-Builder Exercise 7.5 235
The Practice of Forecasting 238
Basic Concepts Review Questions 239
Problems and Applications 240
Case: Energy Forecasting 241

Chapter 8 INTRODUCTION TO STATISTICAL QUALITY CONTROL 248
Introduction 248
The Role of Statistics and Data Analysis in Quality Control 249
Statistical Process Control 250
Control Charts 250
$\bar{x}$- and *R*-Charts 251
Skill-Builder Exercise 8.1 256
Analyzing Control Charts 256
Sudden Shift in the Process Average 257
Cycles 257
Trends 257

Hugging the Center Line 257
Hugging the Control Limits 258
Skill-Builder Exercise 8.2 258
Skill-Builder Exercise 8.3 260
Control Charts for Attributes 260
Variable Sample Size 262
Skill-Builder Exercise 8.4 264
Process Capability Analysis 264
Skill-Builder Exercise 8.5 266
Basic Concepts Review Questions 266
Problems and Applications 266
Case: Quality Control Analysis 267

Part II Decision Modeling and Analysis 269

Chapter 9 BUILDING AND USING DECISION MODELS 271
Introduction 271
Decision Models 272
Model Analysis 275
What-If Analysis 275
Skill-Builder Exercise 9.1 277
Skill-Builder Exercise 9.2 278
Skill-Builder Exercise 9.3 278
Model Optimization 278
Tools for Model Building 280
Logic and Business Principles 280
Skill-Builder Exercise 9.4 281
Common Mathematical Functions 281
Data Fitting 282
Skill-Builder Exercise 9.5 284
Spreadsheet Engineering 284
Skill-Builder Exercise 9.6 285
Spreadsheet Modeling Examples 285
New Product Development 285
Skill-Builder Exercise 9.7 287
Single Period Purchase Decisions 287
Overbooking Decisions 288
Project Management 289
Model Assumptions, Complexity, and Realism 291
Skill-Builder Exercise 9.8 293
Basic Concepts Review Questions 293
Problems and Applications 294
Case: An Inventory Management Decision Model 297

Chapter 10 DECISION MODELS WITH UNCERTAINTY AND RISK 300
Introduction 301
Spreadsheet Models with Random Variables 301
Monte Carlo Simulation 302
Skill-Builder Exercise 10.1 303
Monte Carlo Simulation Using *Crystal Ball* 303
Defining Uncertain Model Inputs 304
Running a Simulation 308
Saving *Crystal Ball* Runs 310
Analyzing Results 310
Skill-Builder Exercise 10.2 314
Crystal Ball Charts 315
Crystal Ball Reports and Data Extraction 318
Crystal Ball Functions and Tools 318
Applications of Monte Carlo Simulation and *Crystal Ball* Features 319
Newsvendor Model: Fitting Input Distributions, *Decision Table* Tool, and Custom Distribution 319
Skill-Builder Exercise 10.3 323
Skill-Builder Exercise 10.4 324
Overbooking Model: *Crystal Ball* Functions 324
Skill-Builder Exercise 10.5 325
Cash Budgeting: Correlated Assumptions 325
New Product Introduction: *Tornado Chart* Tool 328
Skill-Builder Exercise 10.6 329
Project Management: Alternate Input Parameters and the *Bootstrap* Tool 329
Skill-Builder Exercise 10.7 334
Basic Concepts Review Questions 334
Problems and Applications 335
Case: J&G Bank 338

Chapter 11 DECISIONS, UNCERTAINTY, AND RISK 343
Introduction 344
Decision Making Under Certainty 344
Decisions Involving a Single Alternative 345
Skill-Builder Exercise 11.1 345
Decisions Involving Non–mutually Exclusive Alternatives 345
Decisions Involving Mutually Exclusive Alternatives 346
Decisions Involving Uncertainty and Risk 347
Making Decisions with Uncertain Information 347
Decision Strategies for a Minimize Objective 348

Skill-Builder Exercise 11.2 350
Decision Strategies for a Maximize Objective 350
Risk and Variability 351
Expected Value Decision Making 353
Analysis of Portfolio Risk 354
Skill-Builder Exercise 11.3 356
The "Flaw of Averages" 356
Skill-Builder Exercise 11.4 356
Decision Trees 357
A Pharmaceutical R&D Model 357
Decision Trees and Risk 358
Sensitivity Analysis in Decision Trees 360
Skill-Builder Exercise 11.5 360
The Value of Information 360
Decisions with Sample Information 362
Conditional Probabilities and Bayes's Rule 363
Utility and Decision Making 365
Skill-Builder Exercise 11.6 368
Exponential Utility Functions 369
Skill-Builder Exercise 11.7 370
Basic Concepts Review Questions 370
Problems and Applications 371
Case: The Sandwich Decision 375

Chapter 12 QUEUES AND PROCESS SIMULATION MODELING 378
Introduction 378
Queues and Queuing Systems 379
Basic Concepts of Queuing Systems 379
Customer Characteristics 380
Service Characteristics 381
Queue Characteristics 381
System Configuration 381
Performance Measures 382
Analytical Queuing Models 382
Single-Server Model 383
Skill-Builder Exercise 12.1 384
Little's Law 384
Process Simulation Concepts 385
Skill-Builder Exercise 12.2 386
Process Simulation with *SimQuick* 386
Getting Started with *SimQuick* 387
A Queuing Simulation Model 388

Skill-Builder Exercise 12.3 392
Queues in Series with Blocking 393
Grocery Store Checkout Model with Resources 394
Manufacturing Inspection Model with Decision Points 397
Pull System Supply Chain with Exit Schedules 400
Other *SimQuick* Features and Commercial Simulation Software 402
Continuous Simulation Modeling 403
Basic Concepts Review Questions 406
Problems and Applications 407
Case: Production/Inventory Planning 410

Chapter 13 LINEAR OPTIMIZATION 411
Introduction 411
Building Linear Optimization Models 412
Characteristics of Linear Optimization Models 415
Implementing Linear Optimization Models on Spreadsheets 416
Excel Functions to Avoid in Modeling Linear Programs 417
Solving Linear Optimization Models 418
Solving the SSC Model Using Standard Solver 418
Solving the SSC Model Using *Premium Solver* 420
Solver Outcomes and Solution Messages 422
Interpreting *Solver* Reports 422
Skill-Builder Exercise 13.1 426
How *Solver* Creates Names in Reports 427
Difficulties with *Solver* 427
Applications of Linear Optimization 427
Process Selection 429
Skill-Builder Exercise 13.2 430
Blending 430
Skill-Builder Exercise 13.3 432
Portfolio Investment 432
Skill-Builder Exercise 13.4 433
Transportation Problem 433
Interpreting Reduced Costs 437
Multiperiod Production Planning 437
Skill-Builder Exercise 13.5 439
Multiperiod Financial Planning 439
Skill-Builder Exercise 13.6 440
A Model with Bounded Variables 440
A Production/Marketing Allocation Model 445
How *Solver* Works 449
Basic Concepts Review Questions 450

Problems and Applications 450
Case: Haller's Pub & Brewery 457

Chapter 14 INTEGER, NONLINEAR, AND ADVANCED OPTIMIZATION METHODS 458

Introduction 458
Integer Optimization Models 459
A Cutting Stock Problem 459
Solving Integer Optimization Models 460
Skill-Builder Exercise 14.1 462
Integer Optimization Models with Binary Variables 463
Project Selection 463
Site Location Model 464
Skill-Builder Exercise 14.2 467
Computer Configuration 467
Skill-Builder Exercise 14.3 470
A Supply Chain Facility Location Model 470
Mixed Integer Optimization Models 471
Plant Location Model 471
A Model with Fixed Costs 473
Nonlinear Optimization 475
Hotel Pricing 475
Solving Nonlinear Optimization Models 477
Markowitz Portfolio Model 479
Skill-Builder Exercise 14.4 482
Evolutionary Solver for Nonsmooth Optimization 482
Rectilinear Location Model 484
Skill-Builder Exercise 14.5 484
Job Sequencing 485
Skill-Builder Exercise 14.6 488
Risk Analysis and Optimization 488
Combining Optimization and Simulation 491
A Portfolio Allocation Model 491
Using *OptQuest* 492
Skill-Builder Exercise 14.7 500
Basic Concepts Review Questions 500
Problems and Applications 500
Case: Tindall Bookstores 506

Appendix 509

Index 521

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PREFACE

INTENDED AUDIENCE

Statistics, Data Analysis, and Decision Modeling was written to meet the need for an introductory text that provides the fundamentals of business statistics and decision models/optimization, focusing on practical applications of data analysis and decision modeling, all presented in a simple and straightforward fashion.

The text consists of 14 chapters in two distinct parts. The first eight chapters deal with statistical and data analysis topics, while the remaining chapters deal with decision models and applications. Thus, the text may be used for:

- MBA or undergraduate business programs that combine topics in business statistics and management science into a single, brief, quantitative methods
- Business programs that teach statistics and management science in short, modular courses
- Executive MBA programs
- Graduate refresher courses for business statistics and management science

NEW TO THIS EDITION

The fifth edition of this text has been carefully revised to improve clarity and pedagogical features, and incorporate new and revised topics. Many significant changes have been made, which include the following:

1. Spreadsheet-based tools and applications are compatible with *Microsoft Excel 2010*, which is used throughout this edition.
2. Every chapter has been carefully revised to improve clarity. Many explanations of critical concepts have been enhanced using new business examples and data sets. The sequencing of several topics have been reorganized to improve their flow within the book.
3. Excel, *PHStat*, and other software notes have been moved to chapter appendixes so as not to disrupt the flow of the text.
4. "Skill-Builder" exercises, designed to provide experience with applying Excel, have been located in the text to facilitate immediate application of new concepts.
5. Data used in many problems have been changed, and new problems have been added.

SUBSTANCE

The danger in using quantitative methods does not generally lie in the inability to perform the requisite calculations, but rather in the lack of a fundamental understanding of why to use a procedure, how to use it correctly, and how to properly interpret results. A key focus of this text is conceptual understanding using simple and practical examples rather than a plug-and-chug or point-and-click mentality, as are often done in other texts, supplemented by appropriate theory. On the other hand, the text does not attempt to be an encyclopedia of detailed quantitative procedures, but focuses on useful concepts and tools for today's managers.

To support the presentation of topics in business statistics and decision modeling, this text integrates fundamental theory and practical applications in a spreadsheet environment using *Microsoft Excel 2010* and various spreadsheet add-ins, specifically:

- *PHStat*, a collection of statistical tools that enhance the capabilities of Excel; published by Pearson Education

- *Crystal Ball* (including *CBPredictor* for forecasting and *OptQuest* for optimization), a powerful commercial package for risk analysis
- *TreePlan*, a decision analysis add-in
- *SimQuick*, an Excel-based application for process simulation, published by Pearson Education
- *Risk Solver Platform for Education*, an Excel-based tool for risk analysis, simulation, and optimization

These tools have been integrated throughout the text to simplify the presentations and implement tools and calculations so that more focus can be placed on interpretation and understanding the managerial implications of results.

TO THE STUDENTS

The Companion Website for this text (**www.pearsonhighered.com/evans**) contains the following:

- ***Data files***—download the data and model files used throughout the text in examples, problems, and exercises
- ***PHStat***—download of the software from Pearson
- ***TreePlan***—link to a free trial version
- ***Risk Solver Platform for Education***—link to a free trial version
- ***Crystal Ball***—link to a free trial version
- ***SimQuick***—link that will direct you to where you may purchase a standalone version of the software from Pearson
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PART I

Statistics and Data Analysis

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Chapter 1

Data and Business Decisions

- INTRODUCTION 4
- DATA IN THE BUSINESS ENVIRONMENT 4
- SOURCES AND TYPES OF DATA 6
  - Metrics and Data Classification 7
- STATISTICAL THINKING 11
  - Populations and Samples 12
- USING MICROSOFT EXCEL 13
  - Basic Excel Skills 14
  - Copying Formulas and Cell References 14
  - Functions 16
  - Other Useful Excel Tips 18
  - Excel Add-Ins 19
- DISPLAYING DATA WITH EXCEL CHARTS 21
  - Column and Bar Charts 21
  - Line Charts 23
  - Pie Charts 23
  - Area Charts 24
  - Scatter Diagrams 24
  - Miscellaneous Excel Charts 25
  - Ethics and Data Presentation 25
- BASIC CONCEPTS REVIEW QUESTIONS 27
- PROBLEMS AND APPLICATIONS 27
- CASE: A DATA COLLECTION AND ANALYSIS PROJECT 28
- APPENDIX 1.1: EXCEL AND *PHSTAT* NOTES 28
  - A. Using the *PHStat Stack Data* and *Unstack Data* Tools 28
  - B. Creating Charts in Excel 2010 29

INTRODUCTION

Since the dawn of the electronic age and the Internet, both individuals and organizations have had access to an enormous wealth of data and information. *Data* are numerical facts and figures that are collected through some type of measurement process. *Information* comes from analyzing data; that is, extracting meaning from data to support evaluation and decision making. Modern organizations—which include for-profit businesses such as retailers, manufacturers, hotels, and airlines, as well as nonprofit organizations like hospitals, educational institutions, and government agencies—need good data to evaluate daily performance and to make critical strategic and operational decisions.

The purpose of this book is to introduce you to statistical methods for analyzing data; ways of using data effectively to make informed decisions; and approaches for developing, analyzing, and solving models of decision problems. Part I of this book (Chapters 1–8) focuses on key issues of statistics and data analysis, and Part II (Chapters 9–14) introduces you to various types of decision models that rely on good data analysis.

In this chapter, we discuss the roles of data analysis in business, discuss how data are used in evaluating business performance, introduce some fundamental issues of statistics and measurement, and introduce spreadsheets as a support tool for data analysis and decision modeling.

DATA IN THE BUSINESS ENVIRONMENT

Data are used in virtually every major function in business, government, health care, education, and other nonprofit organizations. For example:

- Annual reports summarize data about companies' profitability and market share both in numerical form and in charts and graphs to communicate with shareholders.
- Accountants conduct audits and use statistical methods to determine whether figures reported on a firm's balance sheet fairly represents the actual data by examining samples (that is, subsets) of accounting data, such as accounts receivable.
- Financial analysts collect and analyze a variety of data to understand the contribution that a business provides to its shareholders. These typically include profitability, revenue growth, return on investment, asset utilization, operating margins, earnings per share, economic value added (EVA), shareholder value, and other relevant measures.
- Marketing researchers collect and analyze data to evaluate consumer perceptions of new products.
- Operations managers use data on production performance, manufacturing quality, delivery times, order accuracy, supplier performance, productivity, costs, and environmental compliance to manage their operations.
- Human resource managers measure employee satisfaction, track turnover, training costs, employee satisfaction, turnover, market innovation, training effectiveness, and skills development.
- Within the federal government, economists analyze unemployment rates, manufacturing capacity and global economic indicators to provide forecasts and trends.
- Hospitals track many different clinical outcomes for regulatory compliance reporting and for their own analysis.
- Schools analyze test performance and state boards of education use statistical performance data to allocate budgets to school districts.

Data support a variety of company purposes, such as planning, reviewing company performance, improving operations, and comparing company performance with competitors' or "best practices" benchmarks. Data that organizations use should focus on critical success factors that lead to competitive advantage. An example from the

Boeing Company shows the value of having good business data and analysis capabilities.[1] In the early 1990s, Boeing's assembly lines were morasses of inefficiency. A manual numbering system dating back to World War II bomber days was used to keep track of an airplane's four million parts and 170 miles of wiring; changing a part on a 737's landing gear meant renumbering 464 pages of drawings. Factory floors were covered with huge tubs of spare parts worth millions of dollars. In an attempt to grab market share from rival Airbus, the company discounted planes deeply and was buried by an onslaught of orders. The attempt to double production rates, coupled with implementation of a new production control system, resulted in Boeing being forced to shut down its 737 and 747 lines for 27 days in October 1997, leading to a $178 million loss and a shakeup of top management. Much of the blame was focused on Boeing's financial practices and lack of real-time financial data. With a new Chief Financial Officer and finance team, the company created a "control panel" of vital measures, such as materials costs, inventory turns, overtime, and defects, using a color-coded spreadsheet. For the first time, Boeing was able to generate a series of charts showing which of its programs were creating value and which were destroying it. The results were eye-opening and helped formulate a growth plan. As one manager noted, "The data will set you free."

Data also provide key inputs to decision models. A **decision model** is a logical or mathematical representation of a problem or business situation that can be developed from theory or observation. Decision models establish relationships between actions that decision makers might take and results that they might expect, thereby allowing the decision makers to predict what might happen based on the model. For instance, the manager of a grocery store might want to know how best to use price promotions, coupons, and advertising to increase sales. In the past, grocers have studied the relationship of sales volume to programs such as these by conducting controlled experiments to identify the relationship between actions and sales volumes.[2] That is, they implement different combinations of price promotions, coupons, and advertising (the decision variables), and then observe the sales that result. Using the data from these experiments, we can develop a predictive model of sales as a function of these decision variables. Such a model might look like the following:

$$\text{Sales} = a + b \times \text{Price} + c \times \text{Coupons} + d \times \text{Advertising} + e \times \text{Price} \times \text{Advertising}$$

where *a, b, c, d,* and *e* are constants that are estimated from the data. By setting levels for price, coupons, and advertising, the model estimates a level of sales. The manager can use the model to help identify effective pricing, promotion, and advertising strategies.

Because of the ease with which data can be generated and transmitted today, managers, supervisors, and front-line workers can easily be overwhelmed. Data need to be summarized in a quantitative or visual fashion. One of the most important tools for doing this is **statistics**, which David Hand, former president of the Royal Statistical Society in the UK, defines as *both the science of uncertainty and the technology of extracting information from data*.[3] Statistics involve collecting, organizing, analyzing, interpreting, and presenting data. A **statistic** is a summary measure of data. You are undoubtedly familiar with the concept of statistics in daily life as reported in newspapers and the media; baseball batting averages, airline on-time arrival performance, and economic statistics such as Consumer Price Index are just a few examples. We can easily google statistical information about investments and financial markets, college loans and home mortgage rates, survey results about national political issues, team and individual

[1] Jerry Useem, "Boeing versus Boeing," *Fortune,* October 2, 2000, 148–160.
[2] "Flanking in a Price War," *Interfaces,* Vol. 19, No. 2, 1989, 1–12.
[3] David Hand, "Statistics: An Overview," in Miodrag Lovric, Ed., *International Encyclopedia of Statistical Science,* Springer Major Reference; http://www.springer.com/statistics/book/978-3-642-04897-5, p. 1504.

sports performance, and well, just about anything. To paraphrase Apple, "There's a stat for that!" Modern spreadsheet technology, such as Microsoft Excel, has made it quite easy to apply statistical tools to organize, analyze, and present data to make them more understandable.

Most organizations have traditionally focused on financial and market information, such as profit, sales volume, and market share. Today, however, many organizations use a wide variety of measures that provide a comprehensive view of business performance. For example, the Malcolm Baldrige Award Criteria for Performance Excellence, which many organizations use as a high-performance management framework, suggest that high-performing organizations need to measure results in five basic categories:

1. *Product and process outcomes,* such as reliability, performance, defect levels, service errors, response times, productivity, production flexibility, setup times, time to market, waste stream reductions, innovation, emergency preparedness, strategic plan accomplishment, and supply chain effectiveness.
2. *Customer-focused outcomes,* such as customer satisfaction and dissatisfaction, customer retention, complaints and complaint resolution, customer perceived value, and gains and losses of customers.
3. *Workforce-focused outcomes,* such as workforce engagement and satisfaction, employee retention, absenteeism, turnover, safety, training effectiveness, and leadership development.
4. *Leadership and governance outcomes,* such as communication effectiveness, governance and accountability, environmental and regulatory compliance, ethical behavior, and organizational citizenship.
5. *Financial and market outcomes.* Financial outcomes might include revenue, profit and loss, net assets, cash-to-cash cycle time, earnings per share, and financial operations efficiency (collections, billings, receivables). Market outcomes might include market share, business growth, and new products and service introductions.

Understanding key relationships among these types of measures can help organizations make better decisions. For example, Sears, Roebuck and Company provided a consulting group with 13 financial measures, hundreds of thousands of employee satisfaction data points, and millions of data points on customer satisfaction. Using advanced statistical tools, the analysts discovered that employee attitudes about the job and the company are key factors that predict their behavior with customers, which, in turn, predicts the likelihood of customer retention and recommendations, which, in turn, predict financial performance. As a result, Sears was able to predict that if a store increases its employee satisfaction score by five units, customer satisfaction scores will go up by two units and revenue growth will beat the stores' national average by 0.5%.[4] Such an analysis can help managers make decisions, for instance, on improved human resource policies.

SOURCES AND TYPES OF DATA

Data may come from a variety of sources: internal record-keeping, special studies, and external databases. Internal data are routinely collected by accounting, marketing, and operations functions of a business. These might include production output, material costs, sales, accounts receivable, and customer demographics. Other data must be generated through special efforts. For example, customer satisfaction data are often acquired by mail,

[4] "Bringing Sears into the New World," *Fortune,* October 13, 1997, 183–184.

Internet, or telephone surveys; personal interviews; or focus groups. External databases are often used for comparative purposes, marketing projects, and economic analyses. These might include population trends, interest rates, industry performance, consumer spending, and international trade data. Such data can be found in annual reports, Standard & Poor's Compustat data sets, industry trade associations, or government databases.

One example of a comprehensive government database is FedStats (www.fedstats.gov), which has been available to the public since 1997. FedStats provides access to the full range of official statistical information produced by the Federal Government without having to know in advance which Federal agency produces which particular statistic. With convenient searching and linking capabilities to more than 100 agencies—which provide data and trend information on such topics as economic and population trends, crime, education, health care, aviation safety, energy use, farm production, and more—FedStats provides one location for access to the full breadth of Federal statistical information.

The use of data for analysis and decision making certainly is not limited to business. Science, engineering, medicine, and sports, to name just a few, are examples of professions that rely heavily on data. Table 1.1 provides a list of data files that are available in the *Statistics Data Files* folder on the Companion Website accompanying this book. All are saved in Microsoft Excel workbooks. These data files will be used throughout this book to illustrate various issues associated with statistics and data analysis and also for many of the questions and problems at the end of the chapters. They show but a sample of the wide variety of applications for which statistics and data analysis techniques may be used.

Metrics and Data Classification

A **metric** is a unit of measurement that provides a way to objectively quantify performance. For example, senior managers might assess overall business performance using such metrics as net profit, return on investment, market share, and customer satisfaction. A supervisor in a manufacturing plant might monitor the quality of a production process for a polished faucet by visually inspecting the products and counting the number of surface defects. A useful metric would be the percentage of faucets that have surface defects. For a web-based retailer, some useful metrics are the percentage of orders filled accurately and the time taken to fill a customer's order. **Measurement** is the act of obtaining data associated with a metric. **Measures** are numerical values associated with a metric.

Metrics can be either discrete or continuous. A **discrete metric** is one that is derived from counting something. For example, a part dimension is either within tolerance or out of tolerance; an order is complete or incomplete; or an invoice can have one, two, three, or any number of errors. Some discrete metrics associated with these examples would be the proportion of parts whose dimensions are within tolerance, the number of incomplete orders for each day, and the number of errors per invoice. **Continuous metrics** are based on a continuous scale of measurement. Any metrics involving dollars, length, time, volume, or weight, for example, are continuous.

A key performance dimension might be measured using either a continuous or a discrete metric. For example, an airline flight is considered on time if it arrives no later than 15 minutes from the scheduled arrival time. We could evaluate on-time performance by counting the number of flights that are late, or by measuring the number of minutes that flights are late. Discrete data are usually easier to capture and record, but provide less information than continuous data. However, one generally must collect a larger amount of discrete data to draw appropriate statistical conclusions as compared to continuous data.

TABLE 1.1 Data Files Available on Companion Website

Business and Economics

Accounting Professionals
Atlanta Airline Data
Automobile Quality
Baldrige
Banking Data
Beverage Sales
Call Center Data
Cell Phone Survey
Cereal Data
China Trade Data
Closing Stock Prices
Coal Consumption
Coal Production
Concert Sales
Consumer Price Index
Consumer Transportation Survey
Credit Approval Decisions
Customer Support Survey
Customer Survey
DJIA December Close
EEO Employment Report
Employees Salaries
Energy Production & Consumption
Federal Funds Rate
Gas & Electric
Gasoline Prices
Gasoline Sales
Google Stock Prices
Hatco
Hi-Definition Televisions
Home Market Value
House Sales
Housing Starts
Insurance Survey
Internet Usage
Microprocessor Data
Mortgage Rates
New Account Processing
New Car Sales
Nuclear Power
Prime Rate
Quality Control Case Data
Quality Measurements
Refrigerators
Residential Electricity Data
Restaurant Sales
Retail Electricity Prices
Retirement Portfolio
Room Inspection
S&P 500
Salary Data
Sales Data
Sampling Error Experiment
Science and Engineering Jobs
State Unemployment Rates
Statistical Quality Control Problems
Surgery Infections
Syringe Samples
Treasury Yield Rates
Unions and Labor Law Data
University Grant Proposals

Behavioral and Social Sciences

Arizona Population
Blood Pressure
Burglaries
California Census Data
Census Education Data
Church Contributions
Colleges and Universities
Death Cause Statistics
Demographics
Facebook Survey
Freshman College Data
Graduate School Survey
Infant Mortality
MBA Student Survey
Ohio Education Performance
Ohio Prison Population
Self-Esteem
Smoking & Cancer
Student Grades
Vacation Survey

Science and Engineering

Pile Foundation
Seattle Weather
Surface Finish
Washington, DC, Weather

Sports

Baseball Attendance
Golfing Statistics
Major League Baseball
NASCAR Track Data
National Football League
Olympic Track and Field Data

	A	B	C	D	E
1	2010 J.D. Power and Associates Initial Quality Statistics				
2					
3	Brand	Problems per 100 Vehicles			
4	Acura	86			
5	Audi	111			
6	BMW	113			
7	Buick	114			
8	Cadillac	111			
9	Chevrolet	111			

FIGURE 1.1 Example of Cross-Sectional, Univariate Data (Portion of *Automobile Quality*)

When we deal with data, it is important to understand the type of data in order to select the appropriate statistical tool or procedure. One classification of data is the following:

1. Types of data
   - ***Cross-sectional***—data that are collected over a single period of time
   - ***Time series***—data collected over time
2. Number of variables
   - ***Univariate***—data consisting of a single variable
   - ***Multivariate***—data consisting of two or more (often related) variables

Figures 1.1–1.4 show examples of data sets from Table 1.1 representing each combination from this classification.

Another classification of data is by the type of measurement scale. Failure to understand the differences in measurement scales can easily result in erroneous or misleading analysis. Data may be classified into four groups:

1. Categorical (nominal) data, which are sorted into categories according to specified characteristics. For example, a firm's customers might be classified by their geographical region (North America, South America, Europe, and Pacific); employees might be classified as managers, supervisors, and associates. The categories bear no quantitative relationship to one another, but we usually assign an arbitrary number to each category to ease the process of managing the data and computing statistics. Categorical data are usually counted or expressed as proportions or percentages.

	A	B	C	D	E	F
1	Banking Data					
2						
3	Median	Median Years	Median	Median	Median Household	Average Bank
4	Age	Education	Income	Home Value	Wealth	Balance
5	35.9	14.8	$91,033	$183,104	$220,741	$38,517
6	37.7	13.8	$86,748	$163,843	$223,152	$40,618
7	36.8	13.8	$72,245	$142,732	$176,926	$35,206
8	35.3	13.2	$70,639	$145,024	$166,260	$33,434
9	35.3	13.2	$64,879	$135,951	$148,868	$28,162
10	34.8	13.7	$75,591	$155,334	$188,310	$36,708
11	39.3	14.4	$80,615	$181,265	$201,743	$38,766

FIGURE 1.2 Example of Cross-Sectional, Multivariate Data (Portion of *Banking Data*)